Dartmouth College, Department of Physics and Astronomy

Whitfield Group

QUANTUM COMPUTATION AND QUANTUM INFORMATION RESEARCH

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Quantum Information Science at Dartmouth

$\langle \Psi | W \rangle$ Whitfield
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QUANTUM SIMULATION USING A TRUNCATED TAYLOR SERIES

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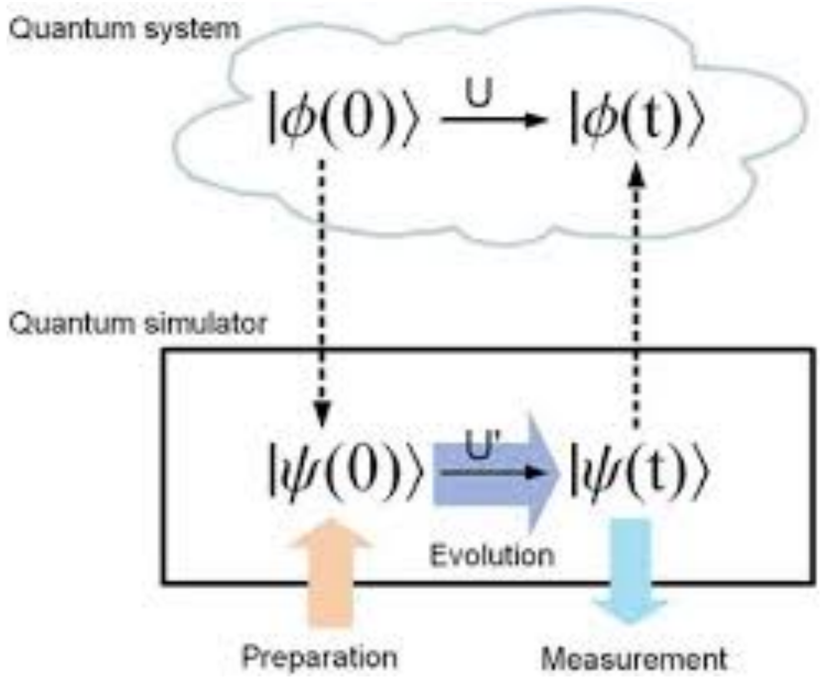

$|\psi(0)\rangle := known$

Hamiltonian simulation

$|\psi(t)\rangle = ?$

Hamiltonian Simulation Problem

Given a description of the Hamiltonian H and evolution time t, perform e^{-iHt} up to some error ε

GENERAL DECOMPOSITION THEORY OF EXPONENTIAL OPERATORS

Masuo Suzuki
Department of Physics, University of Tokyo, Tokyo 113, Japan

First published 1993
First reprint 1998

RESEARCH ARTICLES

Universal Quantum Simulators

Seth Lloyd

Feynman's 1982 conjecture, that quantum computers can be programmed to simulate any local quantum system, is shown to be correct.

SCIENCE • VOL. 273 • 23 AUGUST 1996

LA-UR-14-22745, MIT-CTP #4618

Simulating Hamiltonian dynamics with a truncated Taylor series

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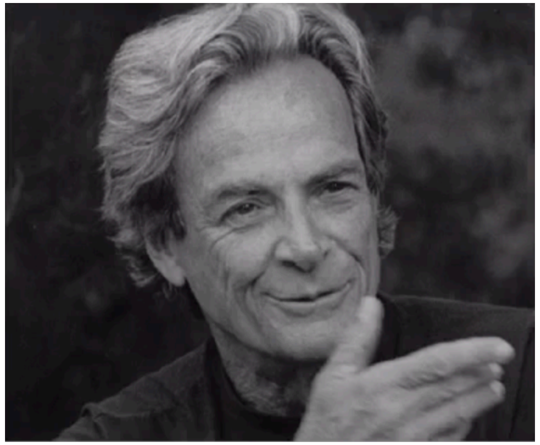
(Dated: December 16, 2014)

We describe a simple, efficient method for simulating Hamiltonian dynamics on a quantum computer by approximating the truncated Taylor series of the evolution operator. Our method can simulate the time evolution of a wide variety of physical systems. As in another recent algorithm, the cost of our method depends only logarithmically on the inverse of the desired precision, which is optimal. However, we simplify the algorithm and its analysis by using a method for implementing linear combinations of unitary operations to directly apply the truncated Taylor series.

PRODUCT FORMULAS VS. TAYLOR SERIES

$$e^{-itH} \approx \prod_{j=1}^{J} \left(e^{-itH_j/r}\right)^r$$

$$e^{-itH} \approx \sum_{k=0}^{K} \frac{(-itH)^k}{k!}$$



"… nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

1982 - Richard Feynman

Spectroscopic signatures of localization with interacting photons in superconducting qubits

P. Roushan[1,*,†], C. Neill[2,†], J. Tangpanitanon[3,†], V. M. Bastidas[3,†], A. Megrant[1], R. Barends[1], Y. Chen[1], Z. Che…
+ See all authors and affiliations

Science 01 Dec 2017:
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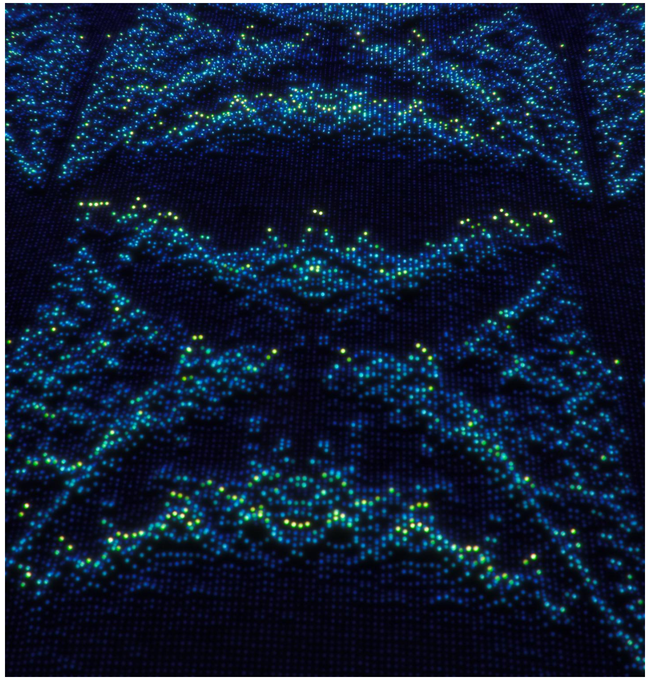
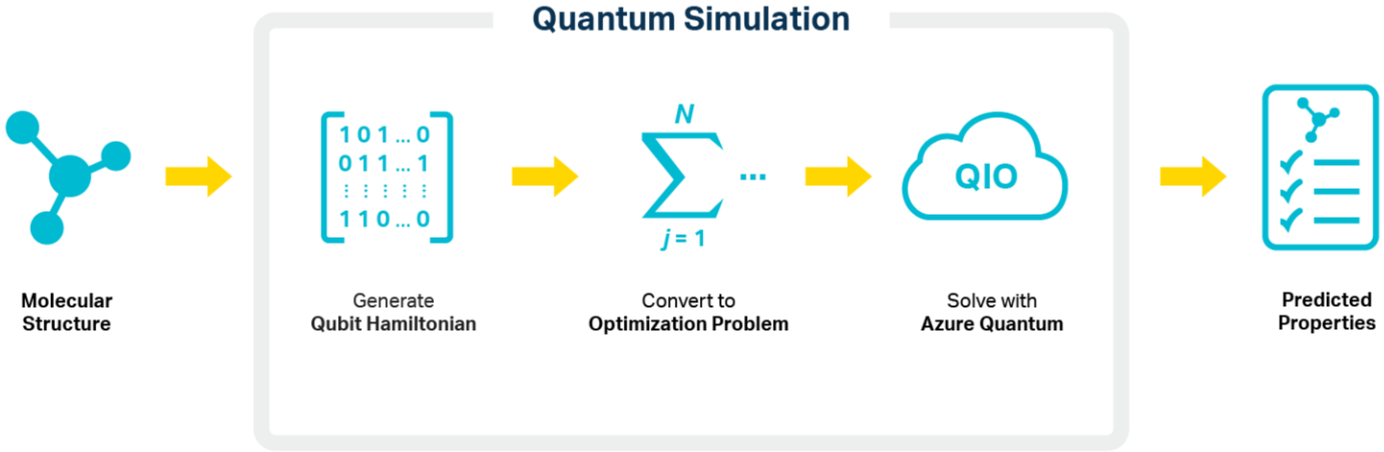
DOI: 10.1126/science.aao1401



Visual Science/Google



Microsoft

Query complexities

❶ Divide and conquer or ❷ Quantum walks	Algorithms for implementing $\exp(-iHt)$	Query complexity (assume $\|H\|_{\max} \leq 1$)
❶ Divide and conquer	Product formulas [BACS07,CK11]	$O\left(d^3 t (dt/\epsilon)^{1/2k}\right)$
❷ Quantum walks	Phase estimation on quantum walks [Chi08,BC10]	$O(dt/\sqrt{\epsilon})$
❶ Divide and conquer	Fractional queries [BCCKS13] or Truncated Taylor series [BCCKS14] (LCU + OAA)	$O\left(d^2 t \frac{\log(d^2 t/\epsilon)}{\log\log(d^2 t/\epsilon)}\right)$
❷ Quantum walks	Linear combination of quantum walks [BCK15] (LCU + OAA)	$O\left(dt \frac{\log(dt/\epsilon)}{\log\log(dt/\epsilon)}\right)$
Lower bound	Shown in [BCCKS13,BCK15]	$\Omega\left(dt + \frac{\log(1/\epsilon)}{\log\log(1/\epsilon)}\right)$
❷ Quantum walks	Quantum signal processing [LC16]	$O(dt + \log(1/\epsilon))$
❶ Divide and conquer	Qubitization/Quantum signal processing [LC17]	$O(d^2 t + \log(1/\epsilon))$

[BACS07] Berry, Ahokas, Cleve, & Sanders. [CK11] Childs & K. [Chi08] Childs. [BC10] Berry & Childs. [BCCKS13,14] Berry, Childs, Cleve, K., & Somma [BCK15] Berry, Childs, & K. [LC16,17] Low & Chuang

$$H\Psi = E\Psi$$

What is Hamiltonian Mechanics?

$$H = \sum_{i=1}^{n} \dot{x}_i p_i - L \quad \text{(Lagrangian)}$$

since $\dot{x}p = v\,p$

$= v\,(mv)$

$= 2\,(\tfrac{1}{2}mv^2)$

$= 2\,KE$

$H = 2KE - (KE - PE)$

$H = KE + PE$

if $PE = f(x) = cx$

then $H = \frac{1}{2}mv^2 + cx$

$= \frac{1}{2m} m^2\dot{x}^2 + cx$

$= \frac{1}{2m} p^2 + cx$

H is Hamiltonian Operator

$$H = \frac{-\hbar^2}{2m}\nabla^2 - V(r)$$