GRADUATE QUALIFYING EXAMINATION

Part I – SHORT QUESTIONS

Dartmouth College
Department of Physics and Astronomy
September 20, 2007 — 9:00 AM to 12:00 noon

Your code number:

INSTRUCTIONS: Answer any 12 out of the 20 questions. Answer in any order you wish, but identify each question by its number as given below. You may use a calculator.

It is suggested that you read all the questions before deciding which to answer.

Please write legibly and make your reasoning as clear as possible.

TABLE OF CONSTANTS

speed of light	$c = 2.99792458 \times 10^8$ m/s
gravitational constant	$G = 6.673(10) \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Planck's constant	$\hbar = 1.054572 \times 10^{-34}$ J s
Boltzmann's constant	$k = 1.3806 \times 10^{-23}$ J/K
permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N}\,\mathrm{A}^{-2}$
permittivity of vacuum	$\epsilon_0 = (c^2\mu_0)^{-1}\,\mathrm{F}\,\mathrm{m}^{-1}$
electron mass	$m_e = 9.109389 \times 10^{-31}$ kg
proton mass	$m_p = 1.672623 \times 10^{-27}$ kg
electron charge	$e = -1.602177 \times 10^{-19}$ C
fine structure constant	$\alpha = e^2/(4\pi\epsilon_0\hbar c) = 1/137.03599$
solar mass	$M_\odot = 1.989 \times 10^{30}$ kg
solar luminosity	$L_\odot = 3.90 \times 10^{26}$ W
solar radius	$R_\odot = 6.96 \times 10^5$ km

1. A spherically symmetric body has a moment of inertia I. What are its lowest three rotational energy levels, and what is the degeneracy of each level?

2. A nonrelativistic particle of mass m and energy E is incident upon a fixed scattering center with which it interacts through a potential of range a. Estimate the number of partial waves that contribute to the scattering amplitude. (OR, Below what energy E does scattering occur predominantly in the S-wave channel?)

3. Make an order of magnitude estimate of the ionization energy of ^{92}U, if the exclusion principle did not operate so that all of its electrons were in the $n = 1$ shell. Assume that the typical electron feels the nuclear charge shielded by the charge of half the other electrons in the shell. The measured ionization energy is $|E| = 4$ eV. Explain the discrepancy.

4. A particle is in a state described by a wavefunction $\psi(x) = Ae^{-ax}$, $a > 0$. Find the length of an interval around the origin such that the probability of measuring the particle's position in this interval is 40%.

5. Consider a system consisting of N noninteracting atoms at temperature T. The atoms are placed in an external magnetic field $\mathbf{H}$ pointing along the z direction. The magnetic moment of the atom is $\mu = g\mu_0\mathbf{J}$, where g is the atom's g-factor, μ_0 is Bohr's magneton, and $\hbar\mathbf{J}$ is the atom's total angular momentum.

   (a) Compute the possible magnetic energies of a given atom.

   (b) If the atoms are in thermal equilibrium at temperature T, what is the probability of finding one of the atoms in a given state?

6. Many systems exist in different phases-one notable example being solid-liquid-gas. Systems undergo phase transitions when parameters such as temperature or pressure are varied. The transition can be either first order or continuous. Please explain what conditions determine a phase transition to be classified as first order or continuous.

7. It is possible to dip a wet (with water) finger into boiling oil for a brief instant, without suffering injury to the finger. This is the so-called Leidenfrost effect. Explain the basic physics behind this effect.

8. Write down the definition of absolute temperature of a system in terms of its total entropy S and total energy E. Suppose the system could have a maximum energy $E_{\text{max}} \geq E$, with only one state at that maximum energy. How would you qualitatively expect the entropy of the system to vary as the system energy approached

E_{max}? What would be the consequence then for the system temperature at energies close to E_{max}?

9. Calculate the radius of the geosynchronous orbit, i.e., the circular orbit in which a satellite remains constantly above a particular point of Earth's equator.

10. Two beads of equal mass m are attached to each other by a spring with Hooke's constant k and constrained to move in one dimension. The equations of motion for the beads are

$$\begin{aligned} m\ddot{x}_1 &= -k(x_1 - x_2) \\ m\ddot{x}_2 &= -k(x_2 - x_1). \end{aligned}$$

Work out the normal mode frequencies for the system and describe qualitatively the motion of the system when it is oscillating with each of the single normal mode frequencies.

11. When the Millenium Suspension Bridge across the River Thames in London was opened on 10 June 2000 by the Queen, it was found to exhibit unacceptably large (and uncomfortable) oscillatory motion when she and her large retinue marched across it for the first time. Using the driven, damped harmonic oscillator as a model, give two different ways in which the bridge could be fixed in order to prevent this resonant motion.

12. A conducting rod of length l rotates at constant angular velocity ω about one end, in a plane perpendicular to a uniform magnetic field B. Obtain the potential difference $|V|$ between the ends of the rod.

13. Small particles might be blown out of the solar system by the radiation pressure of sunlight. Assume the particles are spherical with a radius r, each have a mass density ρ, and that they absorb all the radiation in an area πr^2. The particles are a distance R from the sun, which has a power output (solar luminosity) P_s and mass M_s. What is the radius r of the particles for which the radiation force just balances the gravitational attraction of the sun? Express r in terms of P_s, ρ, c, G (gravitational constant), and M_s. (Hint: your answer should not depend on R.)

14. Find the capacitance between two conducting spheres of radii a and b that are separated by a distance d such that $d \gg b \gg a$. Neglect the influence of either sphere on the surface charge distribution of the other. What is the ratio E_a/E_b of the electric fields at the surfaces of the two spheres?

15. A large parallel-plate capacitor with uniform surface charge density $+\sigma$ on the top plate and $-\sigma$ on the bottom plate is moving with constant speed v parallel to the plates.

    (a) Find the magnetic field between the plates as well as above and below the plates.

    (b) Find the magnetic force per unit area on the upper plate.

    (c) At what speed v would the magnetic force balance the electrical force?

16. Briefly explain two different lines of evidence that point to the conclusion that spiral galaxies have massive halos of dark matter.

17. Draw an H-R diagram, and sketch the evolution of a 2-solar mass star. Point out the salient features, and briefly explain their physical causes. Describe the end-state of this star's evolution.

18. Explain the nature of the 21 cm radiation from neutral hydrogen. Approximately how long is the timescale for spontaneous emission for this transition? Briefly describe its usefulness in astronomy.

19. The massive black hole at the center of the galaxy has a mass of about $3 \times 10^6 M_\odot$. How was this mass determined? What is the size of its event horizon?

GRADUATE QUALIFYING EXAMINATION

Part II – LONG QUESTIONS

Dartmouth College
Department of Physics and Astronomy
September 21, 2007 — 9:00 AM to 12:00 noon

Your code number:

INSTRUCTIONS: Answer any 6 out of the 10 questions. Answer in any order you wish, but identify each question by its number as given below. You may use a calculator.

It is suggested that you read all the questions before deciding which to answer.

Please write legibly and make your reasoning as clear as possible.

TABLE OF CONSTANTS

speed of light	$c = 2.99792458 \times 10^8$ m/s
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1. A particle has spin $\hbar$. A measurement of the spin of this particle along a given direction yields the largest possible value. Next, a measurement of the spin is made along a new direction making an angle θ with the original direction.

   (a) What are the possible results of this measurement?

   (b) What are the probabilities for the results predicted in part (a)?

2. Quantum electromagnetic vacuum fluctuations give rise to an attractive force between the parallel plates of a capacitor, even when there is no net charge on the plates. This is called the Casimir force.

   (a) Assuming that for perfectly conducting parallel plates in vacuum, the Casimir force depends only on Planck's constant $\hbar$, the speed of light c, and the plate separation d, use dimensional analysis to determine (up to a dimensionless constant) the formula for the Casimir force per unit area.

   (b) The actual dimensionless constant is $\pi^2/240$. Suppose the plates are separated by a gap $d = 10^{-6}$ meters (i.e., one micron). How large must the plate area be such that the Casimir force is approximately one Newton?

   (c) An analogous attractive force has been predicted to exist between two parallel plates immersed in liquid helium due to quantum sound wave fluctuations. This is called the acoustic Casimir force. Would you expect this force to be larger or smaller than the electromagnetic Casimir force for the same plate geometry? Why?

3. Two identical spin 1/2 particles occupy neighboring sites. The spin part of their Hamiltonian is described by $H = K\sigma_{1z}\sigma_{2z}$, where σ_{iz} is the Pauli spin matrix for the particle occupying site i and K is a coupling constant.

   (a) Determine the energy eigenvalues, their degeneracies, and associated eigenstates, making sure that your chosen eigenstates have the proper symmetries.

   (b) Suppose that $K < 0$. Which of the eigenstates obtained in part (a) are the ground states?

   (c) Suppose that $K > 0$. Which of the eigenstates obtained in part (a) are the ground states now?

   (d) For which one of the two cases $K < 0$ or $K > 0$ would the ground state be useful for quantum information applications and why?

   (e) The observable known as the magnetization is proportional to $\sigma_{1z} + \sigma_{2z}$. Plot qualitatively the dependence of the average magnetization on temperature for the case $K < 0$ when the system is in thermal contact with a heat bath.

   (f) Repeat part (e) for the case $K > 0$.

   (g) On the basis of your answers for parts (e) and (f), how could you distinguish the two cases $K < 0$ and $K > 0$ by measuring the magnetization?

4. Consider an ideal gas of N bosons of mass m, confined to a volume V and in thermal equilibrium at temperature T.

   (a) Explain what is meant by Bose-Einstein condensation for this system, specifying the momentum (or energy) distribution function for $0 < T < T_c$, where T_c is the critical temperature.

   (b) Write an expression for the internal energy U in terms of an integral involving the distribution function, for $0 < T < T_c$. Do not attempt to evaluate the integral, but show that $U \sim T^n$ and find n.

   (c) Write an expression, again involving an integral of the distribution function, from which one could determine T_c. Obtain T_c in terms of a dimensionless integral.

5. Consider a simple harmonic oscillator of classical angular frequency ω. The oscillator is in contact with a heat bath at temperature T.

   (a) Compute the mean total energy $\bar{E}$.

   (b) Now consider the same oscillator in the quantum regime. Compute the partition function Z.

   (c) Obtain the high-temperature limit of $\bar{E}$ and compare with your answer in a).

   (d) Obtain the low-temperature limit of $\bar{E}$ and take the limit $T \to 0$. Compare your answer to the energy levels of the simple harmonic oscillator.

6. A rigid rod pendulum of length L and uniform mass per unit length M/L has a fixed pivot point at one end and is constrained to swing in a plane about the vertical as defined by the direction of the gravitational force. An additional point mass M is attached to the rigid rod pendulum at some distance $\ell < L$ from its fixed pivot point.

   (a) Determine the Lagrangian of the pendulum and attached point mass in terms of the angular displacement θ from the vertical and angular velocity $\dot{\theta}$.

   (b) From your answer in part (a), derive the equations of motion.

   (c) From your answer in part (b), derive an expression for the angular frequency ω of small oscillations in terms of L, ℓ, and Newton's constant g.

   (d) In the limit of small oscillations, what is ω for the cases $\ell = L$ and $\ell = 0$?

   (e) Determine by trial and error (or otherwise) the approximate ℓ for which ω is a maximum in the limit of small oscillations. What is ω for this case?

7. A pendulum is rigidly fixed to an axle held by two supports so that it is constrained to swing in the plane perpendicular to the axle. The pendulum consists of a mass m and a massless rod of length L. The supports for the pendulum are mounted on

a table that rotates with constant angular velocity Ω. The angle θ measures the angular displacement of the pendulum from the vertical.

(a) Write down expressions for the kinetic and potential energies and from those derive the Hamiltonian function for the pendulum.

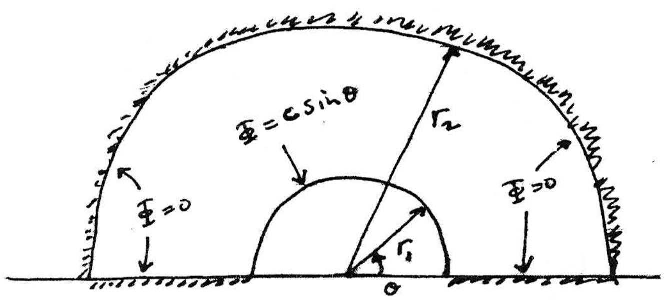
(b) Determine the equation of motion from the canonical equations.

(c) Is H a conserved quantity? Is H equal to the total energy $(T+U)$? Why or why not?

(d) Find any points of equilibrium of the pendulum. For each one, determine whether it is stable or unstable. (Hint: it may be useful to examine the effective potential whose definition should be clear from the form of the Hamiltonian.)

(e) In the case of any stable equilibrium points, determine the frequency of small oscillations about them. (You may choose a more direct method to do this and part (d) if you like.)

8. Consider the following 2D semicircular-shaped region:

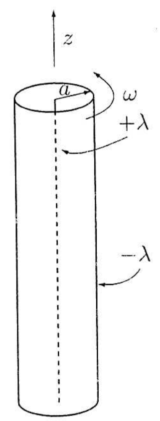


As shown in the figure, the flat parts of the walls at $\theta = 0$ and $\theta = \pi$, as well as the outer circular wall at $r = r_2$, are all held at $\Phi = 0$. The inner circular wall at $r = r_1$ is held at $\Phi = C \sin\theta$ where C is a constant. Use the separation of variables technique to solve the Poisson equation $\nabla^2\Phi = 0$ in the region $r_1 \le r \le r_2$ for arbitrary r_1, r_2, C:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} = 0$$

9. A long solenoid with radius a and n turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\phi}$ direction. Find the electric field (magnitude and direction) at a distance s from the axis, both inside and outside the solenoid, in the quasi-static approximation (ie, neglecting the displacement current in Ampere's law).

10. A very long cylinder of radius a consisting of a central wire surrounded by a dielectric is spun around its axis with angular velocity ω as shown. The inner wire carries a charge per unit length $+\lambda$ while the outer surface of the dielectric carries a charge per unit length $-\lambda$. The dielectric is polarized only very weakly, so that we can treat its dielectric constant as being equal to one. *In the following, be sure to specify the direction of all vector quantities you calculate.*



(a) What is the electric field $\mathbf{E}$ inside the cylinder?

(b) What sheet current density $\mathbf{K}$ is present at the outer surface of the cylinder when it is rotating?

(c) What is the magnetic field $\mathbf{B}$ inside the cylinder when it is rotating?

(d) Using your results for $\mathbf{E}$ and $\mathbf{B}$, find the momentum density $\mathcal{P}_{em}(r)$ and angular momentum density $\ell_{em}(r)$ inside the rotating cylinder associated with the electromagnetic fields.

(e) What additional moment of inertia (per unit length) I_{em} do the electromagnetic fields contribute to the total moment of inertia (per unit length) of the cylinder?

GRADUATE QUALIFYING EXAMINATION

PART II - ASTRONOMY QUESTIONS

Dartmouth College

Department of Physics and Astronomy

September 21, 2007 — 9:00 AM to 12:00 noon

Your Code Number:

INSTRUCTIONS:

Answer any 8 out of the following 11 questions. On the first page of your exam, indicate which questions you are answering and which you are passing over. Do not answer more than 8 questions.

Please make your reasoning as clear as you can, and try to write legibly. You have three hours.

TABLE OF CONSTANTS

speed of light	$c = 2.99792458 \times 10^8$ m/s
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1) Carefully describe Hubble's galaxy classification scheme (the 'tuning fork' diagram). Also, discuss the environments in which the different types of galaxies tend to be found, i.e. the correlation between environment and morphology.

2) Consider a set of N mass points (e.g., stars). The x-components of their velocities are v_{xi}, where $1 \leq i \leq N$.

a) Define the 1-dimensional velocity dispersion.

b) Explain carefully what would be involved in measuring this number in two cases; (1) the x-axis is parallel to our line of sight, and (2) the x-axis is perpendicular to our line of sight. For case (1) distinguish between the cases in which the stars are resolved as individuals and the case in which only the integrated light is available.

c) Explain carefully the physical significance of the velocity dispersion for a bound collection of objects.

d) Give the order of magnitude of the velocity dispersions of (1) a rich cluster of galaxies (in which the mass 'points' are galaxies), (2) an elliptical galaxy, and (3) the stars in the solar neighborhood (say within 100 pc). If necessary, reconcile your answer to (3) with the principle you described in part (c).

3) a) What event or phenomenon caused the universe to become transparent to the background radiation at $z \sim 1000$?

b) What is the ionization state of the intergalactic gas at the present time? How is this determined observationally?

c) Your answers to (a) and (b) can be reconciled only if a certain event transpired in the early universe. Give some piece of observational evidence that constrains the redshift z at which this occurred.

4) Consider an atom or ion that has states at energies E_i, with statistical weights g_i.

a) Suppose that a collection of such atoms are in thermodynamic equilibrium at temperature T. Write an expression that gives the ratio of the populations in any two states with indices j and k.

b) Is this expression likely to be valid of the atoms are in the interstellar medium? Why or why not?

5) Sketch an edge-on view of the visible Milky Way, label the principal parts of the Galaxy (including the different stellar populations) and give a description of each part of the galaxy. In light of these properties, discuss the formation and evolution of the Milky Way.

6) Consider the equation of state $P = K\rho^{1+1/n}$, where P is pressure, ρ is density, K is a constant, as is n. Using this equation of state, derive a single, second order differential equation for the run of density as a function of radius within a star. Clearly state the assumptions you make in your derivation. What values of the index n are astrophysically interesting?

7) How were the elements oxygen and iron created? Provide specific details regarding the physical processes and conditions involved in the creation of these elements.

8) Consider a 30 $M_\odot$ star in which the pressure gradient is given by

$$\frac{dP}{dr} = -\frac{4\pi}{3} G \rho_c^2 r e^{(-r^2/\ell^2)}$$

where ρ_c is the central density, P is the pressure, r is the radius, G is the gravitational constant and $\ell = R/6$, where R is the total radius of the star. Determine pressure, and mass (M_r) as a function of the radius r.

9) a) What are the Friedmann equations?

b) For the "dust" case, qualitatively, how does the scale factor change with time for the three cases $k = 0, +1$, and -1 when $\Lambda = 0$?

c) How is this altered for $\Lambda \neq 0$?

d) How do k and Λ affect the inferred age for the universe?

10) What are the Oort constants A and B?

11) A class of galaxies are spherically symmetric and have a gravitational potential that obeys $\Phi(r) = -GM/\left[b + \sqrt{(b^2 + r^2)}\right]$ (this is known as the *isochrone potential*). Find expressions for the density $\rho(r)$ and circular speed at radius r. Indicate how one calculates the surface brightness $\Sigma(r)$.