GRADUATE QUALIFYING EXAMINATION

PART I - SHORT QUESTIONS

Dartmouth College

Department of Physics and Astronomy

September 14, 2006 — 9:00 AM to 12:00 noon

Your Code Number:

INSTRUCTIONS: Answer any 12 out of the 20 questions. Answer in any order you wish, but identify each question by its number as given below. You may use a calculator.

It is suggested that you read all the questions before deciding which to answer.

Please write legibly and make your reasoning as clear as possible.

TABLE OF CONSTANTS

speed of light	$c = 2.99792458 \times 10^8 \text{ m/s}$
gravitational constant	$G = 6.673(10) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Planck's constant	$\hbar = 1.054572 \times 10^{-34} \text{ J s}$
Boltzmann's constant	$k = 1.3806 \times 10^{-23} \text{ J/K}$
permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$
permittivity of vacuum	$\epsilon_0 = (c^2 \mu_0)^{-1} \text{ F m}^{-1}$
electron mass	$m_e = 9.109389 \times 10^{-31} \text{ kg}$
proton mass	$m_p = 1.672623 \times 10^{-27} \text{ kg}$
electron charge	$e = -1.602177 \times 10^{-19} \text{ C}$
fine structure constant	$\alpha = e^2/(4\pi\epsilon_0 \hbar c) = 1/137.03599$
solar mass	$M_\odot = 1.989 \times 10^{30} \text{ kg}$
solar luminosity	$L_\odot = 3.90 \times 10^{26} \text{ W}$
solar radius	$R_\odot = 6.96 \times 10^5 \text{ km}$

1) Why are neutron stars and metals so resistant to compression?

2) Consider the following family of eigenstates of a spin-J particle,

$$|j, j\rangle, ..., |j, m\rangle, ...|j, -j\rangle,$$

where $\hbar^2 j(j+1)$ and $\hbar m$ are the eigenvalues of J^2, J_z respectively. Is it always true that one may rotate these states into each other? That is, given $|j, m\rangle$, $|j, m'\rangle$, is it always possible to find a unitary rotation operator $U^{(j)}$ such that $|j, m'\rangle = U^{(j)}|j, m\rangle$? Please justify your answer.

3) A three dimensional isotropic harmonic oscillator with frequency ω has energy levels

$$E_{n_1, n_2, n_3} = \hbar\omega(n_1 + n_2 + n_3 + 3/2),$$

where each of the quantum numbers n_1, n_2, n_3 can be zero or a positive integer number. Find the degeneracies of the levels of energy $7\hbar\omega/2$ and $9\hbar\omega/2$. Given that the system is in thermal equilibrium with a heat bath at a temperature T, show that the $9\hbar\omega/2$ level is more populated than the $7\hbar\omega/2$ level if $k_B T$ is larger than $\hbar\omega/\ln(5/3)$.

4) Evaluate the Fermi wavenumber for a system of N spin $1/2$ fermions confined in 2D to a square of size $L \times L$.

5) A ladder of mass M and length L rests against a smooth wall and slides without friction on both the wall and the floor. Assuming the end of the ladder remains in contact with the wall, find the Lagrangian of this system using suitable generalized coordinates.

6) The discovery of the first extrasolar planet around a sun-like star was announced in 1995. Observers measured the line-of-sight velocity of the star 51 Pegasi by measuring the Doppler shift of its spectral lines. The line-of-sight velocity of *the star* was found to vary sinusoidally with a period of 4.2 days and an amplitude of 50 m/s.

    a) Using the values given above, what is the mass of the planet assuming we are in the orbital plane of the system and assuming 51 Pegasi has the same mass as our Sun?

    b) Since we do not know the actual inclination of the orbit, the value calculated in part (a) represents a limit on the mass. Is this a lower limit or an upper limit?

7) A black hole has been characterized (somewhat naively) as an object with such intense gravitational field that not even light can escape from it. Approximately how small should you be to behave as a black hole? (You can consider yourself to be a sphere.)

8) Sketch the radiation pattern for an electric charge accelerated in the direction of its motion. Consider the cases of nonrelativistic and relativistic speeds.

9) Estimate the DeBröglie wavelength (λ) of a typical ^{7}Li atom in an ideal gas of such atoms at temperature T. Next, assuming that the gas is held at a number density of $n \approx 10^{14}$ cm^{-3}, estimate the temperature required for Bose-Einstein condensation to occur.

10) A particle of mass m rests under gravitational attraction on a hard floor. Use the uncertainty principle to estimate its ground-state energy and average elevation above the floor, in terms of m, g, and h.

11) Model a buckeyball (C_{60}) as a hollow spherical shell of radius R and uniform surface density. Derive an expression for its lowest, non-zero rotational energy level in terms of the mass M_C of a single carbon atom, the radius R, and Planck's constant. What is the degeneracy of this level?

12) Given the infinite network of resistors shown in the diagram, find the equivalent resistance between terminals A and B. (see diagram)
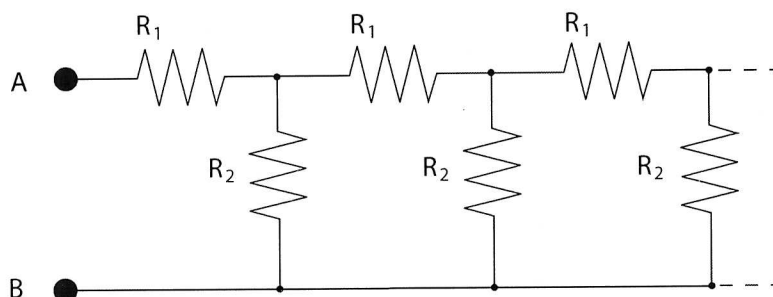
13) A beam of C_{60} molecules is emitted from an oven at temperature T, passes through an interference grating with slit spacing $d = 1$ μm, and is detected on a screen as a spatial interference pattern in the molecule density. Because the internal vibrational modes of the molecules are excited, they radiate photons as they travel though the grating. Explain why the interference pattern disappears as the oven temperature is increased. Estimate the characteristic temperature T of the oven above which the pattern disappears.

14) Is the collapse of the wavefunction a real physical process, or is it just a prescription for calculating the probability of a measurement outcome? Give your point of view in a few sentences.

15) A conducting loop of area A and resistance R is suspended by a torsion spring of constant κ in a uniform magnetic field $\vec{B} = B_0\hat{y}$. The loop can rotate around the z-axis, but lies in the yz plane at equilibrium. The loop has moment of inertia I. The loop is displaced from equilibrium by a small angle ϕ and released. Neglect self-inductance of the loop and assume the torsion spring is non-conducting.

a) What is the equation of motion for the loop?

b) Describe the motion qualitatively (a graph may help).
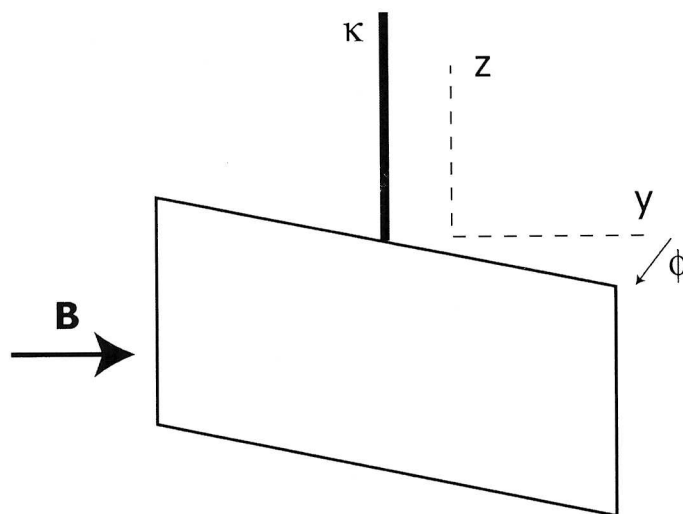
16) Give two pieces of evidence for dark matter in the universe. What are some of the possible candidates for dark matter?

17) Why is the number of solar neutrinos measured by Davis et al at the Homestake Mine experiment only 1/3 of the amount expected?

18) A star that has absolute magnitude $M = +5.8$ has an apparent magnitude of $m = +11.3$.

a) What is its distance modulus?

b) What is its distance in parsecs?

c) If it is on the main sequence, then what kind of star is it?

19) An object is discovered in the solar system at a distance of 100 AU from the sun with a radius of 500 km.

a) What is its orbital period?

b) Would you expect the object to be round? Explain?

20) Ten department colloquia from the past year are listed below. For any *two*, briefly summarize the important issues and outstanding questions pertaining to the topic. Please be as quantitative and specific as possible.

- Being Bayesian in a Quantum World

- Single Molecule Transistors

- Terrestrial Gamma-Ray Flashes

- Itinerant Ferromagnetism: Steps Towards a New Understanding

- Black Holes, Neutron Stars and Supercomputers

- Brownian Motion, Active Fluctuations, and Anomalous Diffusion

- Simulating Physics with a Quantum Computer

- Gravity Tests and the Pioneer Anomaly

- Exploring TeV-scale Physics with the ATLAS Experiment at the Large Hadron Collider

- Trapping Atoms without Laser Cooling: A General Method to Rein in Troublesome Atoms

GRADUATE QUALIFYING EXAMINATION

PART II - LONG QUESTIONS

Dartmouth College

Department of Physics and Astronomy

September 15, 2006 — 9:00 AM to 12:00 noon

Your Code Number:

INSTRUCTIONS: Answer any 6 out of the 10 questions. Answer in any order you wish, but identify each question by its number as given below. You may use a calculator.

It is suggested that you read all the questions before deciding which to answer.

Please write legibly and make your reasoning as clear as possible.

TABLE OF CONSTANTS

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1) Consider a system of N non-interacting, distinguishable particles in which the energy of each particle can assume two distinct values: 0 and ϵ ($\epsilon > 0$). The total energy of the system is U.

a) Derive an expression for the entropy in terms of U, N, ϵ.

b) Sketch the dependence of the entropy S on U.

c) Explain why and where the temperature of the system can be negative.

d) What happens when a system of negative temperature is allowed to exchange heat with a system of positive temperature?

2) Consider a double pendulum as shown below.
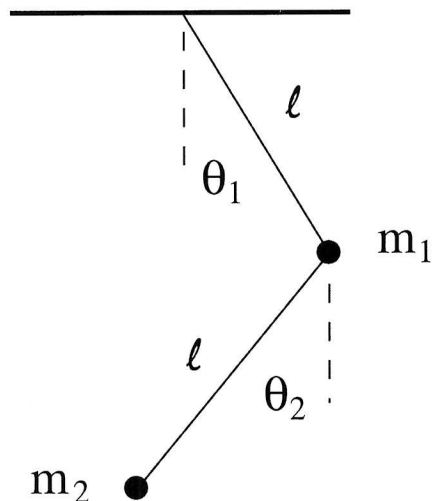
Assume that the length ℓ of each pendulum is the same but that the masses are different. Find the resonant frequencies corresponding to to the normal modes of the oscillation for the system for small values of θ_1 and θ_2. Show that if the mass of the lower pendulum is much smaller than that of the upper one, the frequencies are nearly equal.

3) The relativistic kinetic energy of a particle of mass m and momentum p is

$$K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2.$$

For small momentum, $p \ll mc$, the leading term is $K \approx p^2/(2m)$. Find the next term, which is the leading relativistic correction to the kinetic energy. Use this result to evaluate the correction to the energy for a particle in the ground state of a one-dimensional harmonic oscillator.

4) (a) Calculate the partition function of a classical ideal gas of N atoms in a volume V, in equilibrium with a thermal bath at temperature T. A factor of $N!$ should be included in the denominator - explain the physical basis of this factor.

(b) Calculate the chemical potential and show that it depends only on the temperature and number density of atoms, $n = N/V$. (Recall Stirling's approximation, $\ln N! = N \ln N - N$.)

(c) Assume that this gas is in equilibrium with a solid consisting of the same kinds of atoms. Equate the chemical potential of the gas with that of the solid, assuming that the latter is a constant, $-B$ (with $B > 0$), which may be considered as an effective binding energy per particle. Calculate the resulting density of the gas atoms, n, as a function of T and B, and calculate the pressure.

5) Consider a set of atoms of mass M distributed along a line at points separated by a distance a. Consider longitudinal displacements of the atoms from their equilibrium position. For simplicity assume that only neighboring atoms interact such that you can use a model in which each atom is coupled to its neighbors by perfect springs of sping constant K. Derive the dispersion relation for this system. What range of wavevectors k is physically significant for normal modes?

6) Consider an electron in an eigenstate specified by quantum numbers $|n, \ell, m\rangle$ in a Hydrogen atom.

a) Compute the current density in the azimuthal direction $\hat{e}_\varphi$.

b) Use this result to evaluate the z-component of the magnetic moment of the electron due to its orbital motion.

c) If a static magnetic field $\vec{B} = B\hat{e}_z$ is applied to the atom, what are the energy levels of the system?

7) A non-relativistic electron moves in a region above a large, flat, grounded conductor. The electron is attracted by its image charge, but cannot penetrate the conductor's surface.

(a) Write down the appropriate Hamiltonian for the 3-D motion of the electron. What boundary conditions must the electron's wave function satisfy?

(b) Find the ground state energy, and average distance of the electron above the conductor.

Useful integral:

$$\int_0^\infty du\, u^n\, e^{-u} = n!$$

8) The following, Fokker-Planck (F-P) equation describes the classical statistical dynamics of a harmonic oscillator with frequency ω and mass m in thermal contact with a heat bath at temperature T:

$$\partial\rho/\partial t = \left[\omega^2 x \frac{\partial}{\partial v} - v \frac{\partial}{\partial x} + \gamma \frac{\partial}{\partial v} v + \frac{\gamma k_B T}{m} \frac{\partial^2}{\partial v^2}\right]\rho,$$

where γ is the damping rate and $\rho(x, v, t)dxdv$ gives the probability to find the oscillator with position in the interval $(x, x + dx)$ and velocity in the interval $(v, v + dv)$ at time instant t.

a) From this F-P equation, derive the coupled equations of motion for the three second-moments $\langle x^2 \rangle(t)$, $\langle v^2 \rangle(t)$, and $\langle xv \rangle(t)$, where, for example,

$$\langle x^2 \rangle(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dxdv \; x^2 \rho(x, v, t),$$

etc. (Hint: integration by parts is a useful tool; you may assume that $\rho(x, v, t)$ vanishes for large $|x|$, $|v|$.)

b) Solve for the 2nd moments in the steady-state (e.g., $d\langle x^2 \rangle/dt = 0$, etc.), and verify that your solutions coincide with those derived simply using the Principle of Equipartition.

9) A perfectly conducting fluid contained in a long, cylindrical glass pipe of radius a carries a uniform current I parallel to the axis of the cylinder. In this problem, you will find the electromagnetic contribution to the fluid pressure.

a) Find the electric and magnetic fields everywhere in space.

b) Calculate the electromagnetic energy density inside and outside the cylinder.

c) Find the pressure exerted on the walls of the cylinder.

10) Consider a thin spherical shell of dielectric which has radius R and rotates with angular velocity ω. A constant surface charge of density σ is placed on the sphere. The resulting magnetic field, proportional to ω, is uniform inside the shell, and spatially-dependent outside the shell.

a) Find expressions for the magnetic field both inside and outside the rotating shell.

b) A constant torque $\vec{N}$ is applied parallel to ω. Assuming that the mass of the shell is negligible, hhow long does it take for the shell to stop?

GRADUATE QUALIFYING EXAMINATION

PART TWO – ASTRONOMY QUESTIONS

Dartmouth College

Department of Physics and Astronomy

September 15, 2006 – 9:00am to 12:00 noon

Your Code Number:

Instructions: Answer ten of the twelve questions. Be sure to indicate clearly the question you are answering. You may use a calculator.

Please make your reasoning as clear as you can, and try to write legibly. If you wish to omit something you have written, be sure it is crossed out clearly. You have three hours.

Some of the questions will be rather open-ended and could take a long time to answer in full detail. In view of the time limit, it is best not to be too elaborate.

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1) Describe as specifically as you can either one of the two major reaction pathways for hydrogen fusion in stellar interiors (you don't have to get all the details right, but you should be fairly close). Explain why neutrinos are inevitably produced, in terms of the physics of the relevant interaction.

2) A peculiar galaxy appears spherically symmetric and rotationally supported. The circular orbit velocity in the galaxy follows $V_{circ} \propto r^{1/2}$
Find the run of density in the galaxy. Then, contrast the behavior of this hypothetical object to the behavior of real galaxies.

3) Explain what is meant by the term *Local thermodynamic equilibrium.*
Give one or two examples of formulae that hold true in LTE, but not otherwise (or at least not usually); it'd be good if you could quote the formulae accurately, but don't worry too much if you don't remember them exactly. Finally, when LTE does *not* hold, what kind of computation must be done?

4) (a) Compute the Kelvin-Helmholtz (thermal) timescale for the sun. Compare this to the known age of the sun, and comment on the implications. (b)Explain briefly and quantitatively why the outer part of the sun is convective.

5) Graph a schematic representation of interstellar extinction from the UV (100 nm) to the near-IR (1000 nm). The ratio between visual extinction, A_v, and color excess, $E(B-V)$, is roughly constant; what is that constant? Write down an expression for interstellar extinction as a function of grain size and refractive index. What size dust grains are most responsible for extinction in the UV?

6) Sketch the evolution of the sun from the zero-age main sequence to the white dwarf stage. Describe the major phases and include their timescales.

7) Discuss the Jeans mass equation. (A) Calculate the Jeans mass for typical values for the density and temperature of a typical interstellar neutral hydrogen cloud that could lead to star formation. (B) What are the typical density and temperature values for the various phases of the interstellar medium? What fraction of our galaxy's mass lies in its gas and dust?

8) The inner solar system is comprised of two main types of planets: terrestrial and Jovian. The terrestrial planets have nickel-iron rich cores and lighter element rich mantles and crust. By what mechanism and through what energy/heat source was this element separation accomplished? Over what timescale was this done and is this process active today?

9) What is the CMB C_ℓ harmonic spectrum? Sketch what it looks like and what cosmological information do different features in its structure give?

10) For the flat matter dominated (Einstein-de sitter) universe, show that the age is $t_0 = 2 / (3H_0)$ and the particle horizon distance is $d_{hor}(t_0) = 3ct_0$.

11) What is dynamical friction? Give three examples of processes where it is important.

12) Outline the basic ideas of the spiral density wave theory. What are the physical processes and approximations that go into it? How is the structure of a galaxy affected and how are the density waves observable?